

Math 240 - Quiz 6

March 3, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 8.

1. (5 points) A 4-kg mass is attached to a spring with spring constant 4N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and pushed to the right with an initial speed of 1 m/sec. Find the equation of motion. Write your final result in terms of a single trig function with phase shift.



$x = 0$
(Equilibrium)

$$4x'' + x' + 4x = 0$$

$$x(0) = -1, \quad x'(0) = 1$$

CHAR. EQN: $4r^2 + r + 4 = 0$

$$r = \frac{-1 \pm \sqrt{1 - 4(4)(4)}}{2(4)} =$$

$$\frac{-1 \pm \sqrt{63}i}{8} = -\frac{1}{8} \pm \frac{3\sqrt{7}}{8}i$$

$$x(t) = c_1 e^{-t/8} \cos \frac{3\sqrt{7}}{8} t + c_2 e^{-t/8} \sin \frac{3\sqrt{7}}{8} t$$

$$x(0) = -1 \Rightarrow c_1 = -1$$

$$x'(0) = 1 \Rightarrow -\frac{1}{8}c_1 + \frac{3\sqrt{7}}{8}c_2 = 1$$

$$\Rightarrow \frac{1}{8} + \frac{3\sqrt{7}}{8}c_2 = 1$$

$$\frac{3\sqrt{7}}{8}c_2 = \frac{7}{8} \Rightarrow c_2 = \frac{\sqrt{7}}{3}$$

$$x(t) = -e^{-t/8} \cos \frac{3\sqrt{7}}{8} t + \frac{\sqrt{7}}{3} e^{-t/8} \sin \frac{3\sqrt{7}}{8} t$$

$$A = \sqrt{(-1)^2 + \left(\frac{\sqrt{7}}{3}\right)^2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$$\left. \begin{aligned} \frac{4}{3} \sin \phi &= -1 \\ \frac{4}{3} \cos \phi &= \frac{\sqrt{7}}{3} \end{aligned} \right\} \phi \text{ IN QUAD. IV}$$

$$\phi = \tan^{-1}\left(-\frac{3}{\sqrt{7}}\right)$$

$$x(t) = \frac{4}{3} e^{-t/8} \sin\left(\frac{3\sqrt{7}}{8} t + \tan^{-1}\left(-\frac{3}{\sqrt{7}}\right)\right)$$

Turn over.

2. (5 points) Solve: $y'' - y' - 12y = 2x + 1 + 5e^x$

Homog. eqn: $y'' - y' - 12y = 0$

Char. eqn: $r^2 - r - 12 = 0$

$$(r-4)(r+3) = 0$$

$$r = 4, r = -3$$

$$y_h(x) = c_1 e^{4x} + c_2 e^{-3x}$$

Nonhomog. #1: $g(x) = 2x + 1$

$$y_{p1}(x) = Ax + B$$

$$y'_{p1}(x) = A$$

$$y''_{p1}(x) = 0$$

$$0 - A - 12(Ax + B) = 2x + 1$$

$$-12A = 2 \Rightarrow A = -\frac{1}{6}$$

$$-A - 12B = 1 \Rightarrow B = -\frac{5}{72}$$

$$y_{p1}(x) = -\frac{1}{6}x - \frac{5}{72}$$

Nonhomog. #2: $g(x) = 5e^x$

$$y_p(x) = Ae^x$$

$$y'_p(x) = Ae^x$$

$$y''_p(x) = Ae^x$$

$$Ae^x - Ae^x - 12Ae^x = 5e^x$$

$$-12A = 5 \Rightarrow A = -\frac{5}{12}$$

$$y_{p2}(x) = -\frac{5}{12}e^x$$

$$y(x) = c_1 e^{4x} + c_2 e^{-3x} - \frac{5}{12}e^x - \frac{1}{6}x - \frac{5}{72}$$