

Math 240 - Quiz 6

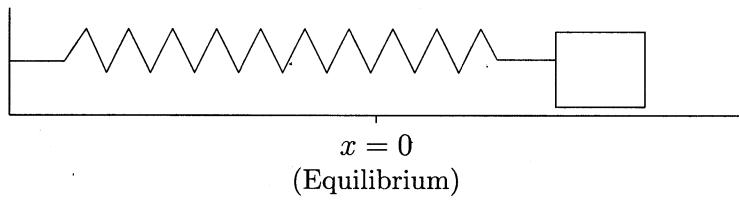
March 3, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 8.

1. (5 points) A 4-kg mass is attached to a spring with spring constant 4 N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and pushed to the right with an initial speed of 1 m/sec. Find the equation of motion. Write your final result in terms of a single trig function with phase shift.



$$4x'' + x' + 4x = 0$$

$$x(0) = -1, \quad x'(0) = 1$$

$$\text{CHAR. EQUATION: } 4r^2 + r + 4 = 0$$

$$r = \frac{-1 \pm \sqrt{1 - 4(4)(4)}}{2(4)} =$$

$$\frac{-1 \pm \sqrt{63}i}{8} = -\frac{1}{8} \pm \frac{3\sqrt{7}}{8}i$$

$$x(t) = C_1 e^{-t/8} \cos \frac{3\sqrt{7}}{8} t + C_2 e^{-t/8} \sin \frac{3\sqrt{7}}{8} t$$

$$x(0) = -1 \Rightarrow C_1 = -1$$

$$x'(0) = 1 \Rightarrow -\frac{1}{8}C_1 + \frac{3\sqrt{7}}{8}C_2 = 1$$

$$\Rightarrow -\frac{1}{8} + \frac{3\sqrt{7}}{8}C_2 = 1$$

$$\frac{3\sqrt{7}}{8}C_2 = \frac{7}{8} \Rightarrow C_2 = \frac{\sqrt{7}}{3}$$

$$x(t) = -e^{-t/8} \cos \frac{3\sqrt{7}}{8} t + \frac{\sqrt{7}}{3} e^{-t/8} \sin \frac{3\sqrt{7}}{8} t$$

$$A = \sqrt{(-1)^2 + \left(\frac{\sqrt{7}}{3}\right)^2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$$\frac{4}{3} \sin \phi = -1 \quad \left. \begin{array}{l} \phi \text{ in} \\ \text{Quadrant IV} \end{array} \right\}$$

$$\frac{4}{3} \cos \phi = \frac{\sqrt{7}}{3}$$

$$\phi = \tan^{-1} \left(-\frac{3}{\sqrt{7}} \right)$$

$$x(t) = \frac{4}{3} e^{-t/8} \sin \left(\frac{3\sqrt{7}}{8} t + \tan^{-1} \left(-\frac{3}{\sqrt{7}} \right) \right)$$

Turn over.

2. (5 points) Solve: $y'' - y' - 12y = 2x + 1 + 5e^x$

Homo. eqn: $y'' - y' - 12y = 0$

Char. eqn: $r^2 - r - 12 = 0$

$$(r-4)(r+3) = 0$$

$$r=4, r=-3$$

$$y_h(x) = c_1 e^{4x} + c_2 e^{-3x}$$

NonHomo. #1: $g(x) = 2x + 1$

$$y_{p1}(x) = Ax + B$$

$$y'_{p1}(x) = A$$

$$y''_{p1}(x) = 0$$

$$0 - A - 12(Ax + B) = 2x + 1$$

$$-12A = 2 \Rightarrow A = -\frac{1}{6}$$

$$-A - 12B = 1 \Rightarrow B = -\frac{5}{72}$$

$$y_{p1}(x) = -\frac{1}{6}x - \frac{5}{72}$$

NonHomo. #2: $g(x) = 5e^x$

$$y_p(x) = Ae^x$$

$$y'_p(x) = Ae^x$$

$$y''_p(x) = Ae^x$$

$$Ae^x - Ae^x - 12Ae^x = 5e^x$$

$$-12A = 5 \Rightarrow A = -\frac{5}{12}$$

$$y_{p2}(x) = -\frac{5}{12}e^x$$

$$y(x) = c_1 e^{4x} + c_2 e^{-3x} - \frac{5}{12}e^x$$

$$-\frac{1}{6}x - \frac{5}{72}$$