Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 5.

1. (5 points) State the recurrence relation that describes the coefficients of the power series solution (centered at $x = 0$), and write the first eight terms of the power series solution.

$$(1 + x)y' - 2y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$0 = \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=1}^{\infty} n a_n x^n - 2 \sum_{n=0}^{\infty} a_n x^n$$

REPLACE n WITH $n+1$

$$= \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=1}^{\infty} n a_n x^n - 2 \sum_{n=0}^{\infty} a_n x^n$$

↑ SAME IF WE START AT $n = 0$

$$= \sum_{n=0}^{\infty} \left[(n+1) a_{n+1} + (n-2) a_n \right] x^n$$

$$y(x) = a_0 + 2a_0 x + a_0 x^2 + 0x^3 + 0x^4 + \cdots$$

$$(n+1) a_{n+1} + (n-2) a_n = 0$$

$$\boxed{a_{n+1} = \frac{(-1)(n-2)}{n+1} a_n; \quad n = 0, 1, 2, \ldots}$$

$$\boxed{y(x) = a_0 \left(1 + 2x + x^2 \right)}$$

$a_0 = $ ARBITRARY

$a_2 = \frac{1}{2} a_1 = a_0$

$a_1 = 2a_0$

$a_3 = 0, \quad a_4 = a_5 = a_6 = \cdots = 0$

Turn over.

2. (5 points) State the recurrence relation that describes the coefficients of the power series solution (centered at $x = 0$), and write the first eight terms of the power series solution.

$$y' + x^3 y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}.$$

$$0 = \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^{n+3}$$

REPLACE
n WITH n+1

REPLACE n
WITH n-3

$$= \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=3}^{\infty} a_{n-3} x^n$$

$$= a_1 + 2 a_2 X + 3 a_3 X^2 + \sum_{n=3}^{\infty} \left[(n+1) a_{n+1} + a_{n-3} \right] x^n$$

$$(n+1) a_{n+1} + a_{n-3} = 0; \quad n = 3, 4, 5, \ldots$$

$a_0 = $ ARBITRARY

$a_1 = 0$

$a_2 = 0$

$a_3 = 0$

$$a_{n+1} = \frac{-1}{n+1} a_{n-3}; \quad n = 3, 4, 5, \ldots$$

$$a_4 = \frac{-1}{4} a_0$$

$a_5 = 0$

$a_6 = 0$

$a_7 = 0$

$$a_8 = -\frac{1}{8} a_4 = \frac{1}{4} \cdot \frac{1}{8} a_0$$

COUNTING THE ZERO TERMS,
THIS IS MORE THAN
8 TERMS.

$$y(x) = a_0 \left(1 - \frac{1}{4} X^4 + \frac{1}{4} \cdot \frac{1}{8} X^8 \right.$$
$$\left. - \frac{1}{4 \cdot 8 \cdot 12} X^{12} + \frac{1}{4 \cdot 8 \cdot 12 \cdot 16} X^{16} \right.$$
$$\left. - \cdots \right)$$