# Math 240 - Quiz 9

April 7, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 12.

1. (6 points) Find two linearly independent power series solutions (centered at $x = 0$). State their recurrence relations, and write the first three terms of each.

$$(1 - x^2)y'' - 2xy' + 12y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$0 = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) a_n x^n - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 12 a_n x^n$$

Replace $n$ with $n+2$ | Same starting w/ $n=0$ | Same starting w/ $n=0$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1) a_n x^n - \sum_{n=0}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 12 a_n x^n$$

$$= \sum_{n=0}^{\infty} \left[ (n+2)(n+1) a_{n+2} + \underbrace{(12 - 2n - n(n-1))}_{-(n+4)(n-3)} a_n \right] x^n$$

$$(n+2)(n+1) a_{n+2} - (n+4)(n-3) a_n = 0$$

$$a_{n+2} = \frac{(n+4)(n-3)}{(n+2)(n+1)} a_n$$

$y(x) = c_1 y_1(x) + c_2 y_2(x)$

$a_0 = 1, \ a_1 = 0$

$a_2 = \frac{-12}{2} a_0 = -6$

$a_3 = 0$

$a_4 = \frac{-6}{12} a_2 = \frac{36}{12} = 3$

$y_1(x) = 1 - 6x^2 + 3x^4 + \dots$

$a_0 = 0, \ a_1 = 1$

$a_2 = 0$

$a_3 = \frac{-10}{6} a_1 = -\frac{5}{3}$

$a_4 = 0$

$a_5 = 0$

All other $a$'s $= 0$

$y_2(x) = x - \frac{5}{3} x^3$

*Turn over.*

2. (1 point) Refer back to problem 1. Determine the radius of convergence that is guaranteed for the series solutions by the theorem in section 3.2.

$$y'' - \frac{2x}{1-x^2} y' + \frac{12}{1-x^2} y = 0$$

SINGULAR PTS ARE $x = \pm 1$

DISTANCE FROM $\pm 1$ TO $0$ IS 1 UNIT

R IS AT LEAST 1.

INTERVAL OF CONVERGENCE IS AT LEAST $(-1, 1)$.

3. (3 points) Use the definition of the Laplace transform to find the transform of $f(t) = 2e^{-8t}$.

$$F(s) = \int_0^\infty e^{-st} 2e^{-8t}\, dt = \int_0^\infty 2e^{-(s+8)t}\, dt$$

$$= -\frac{2}{s+8} e^{-(s+8)t} \Big|_0^\infty = \frac{2}{s+8} e^{-(s+8)t} \Big|_{t\to\infty}^{t=0}$$

$$= \frac{2}{s+8} - \frac{2}{s+8} \cancel{\lim_{t\to\infty} e^{-(s+8)t}}^{\,0 \text{ ASSUMING } s>-8}$$

$$= \boxed{\frac{2}{s+8},\quad s > -8}$$