

Math 240 - Test 2

March 10, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. All integration must be done by hand, unless otherwise specified.

1. (8 points) Solve the following initial value problem.

$$2y'' + 12y' + 18y = 0; \quad y(0) = 2, \quad y'(0) = -1$$

2. (6 points) Find the general solution: $y''' + 3y' = 0$

3. (15 points) Consider the equation $xy'' + y' = 0$, $x > 0$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = \ln x$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Now consider the nonhomogeneous equation $xy'' + y' = 12x^{-3}$, $x > 0$. Verify that $y_p(x) = 3x^{-2}$ is a solution.

(d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $xy'' + y' = 12x^{-3}$; $y(1) = 6$, $y'(1) = 0$.

(e) Is your solution in part (d) unique? Explain.

4. (8 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

(a) $3x'' + 2x' + 1x = 0$

(b) $x(t) = 8e^{-t/5} - 6e^{-2t}$

(c) $3x'' + 3x' + \frac{3}{4}x = 0$

(d) $x(t) = 3 \cos \sqrt{2}t + 2 \sin \sqrt{2}t$

5. (13 points) Find the general solution: $y'' - 7y' + 10y = 9e^{2x}$

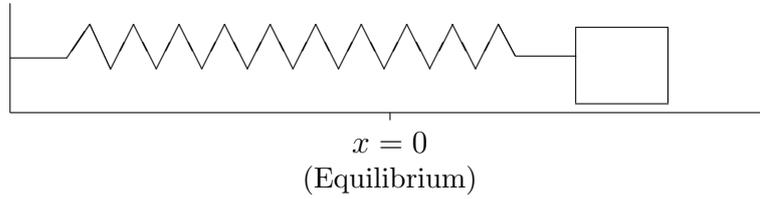
6. (6 points) Consider the equation $yy'' = 6x^4$. Show that $y(x) = x^3$ is a solution, and show that $y(x) = 2x^3$ is NOT a solution. Why is a linear combination of solutions NOT a solution?

7. (8 points) Consider the following equation:

$$y'' - y = x^2e^x + \sin x.$$

Solve the corresponding homogeneous equation. Then use your table to find the appropriate form of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

8. (16 points) A 2-kg mass is attached to a spring with spring constant 6 N/m. The damping constant for the system is 4 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and released from rest. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



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The following problems make up the take-home portion of the test. These problems are due March 22, 2022. You must work on your own.

9. (8 points) Solve the following Cauchy-Euler equation.

$$4x^2y'' - 4xy' + 3y = 0, \quad x > 0$$

10. (12 points) The variation of parameters method that we discussed applies to general 2nd-order linear equations (not only constant-coefficient equations). Use variation of parameters to solve the following nonhomogeneous, Cauchy-Euler equation. Notice that you solved the corresponding homogeneous equation in the previous problem. (Hint: You must divide by $4x^2$ before you apply the method.)

$$4x^2y'' - 4xy' + 3y = 8x^{4/3}, \quad x > 0$$