

Math 240 - Test 3a
April 14, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use a power series centered at $x = 0$ to solve the following equation.

$$(2x + 1)y' - 6y = 0$$

2. (12 points) Use variation of parameters to solve $y'' + 4y = \csc 2x$.

3. (10 points) For each equation below, consider a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Determine the minimum radius of convergence that is guaranteed by the theorem we discussed in class.

(a) $(x + 6)y'' + 7y' + (2x + 3)y = 0$

(b) $e^x y'' + (\cos x)y' - y = 0$

(c) $(x^2 + 3)y'' - 5xy' + (x^2 - 1)y = 0$

4. (6 points) Use the definition of the Laplace transform to find the transform of f .

$$f(t) = \begin{cases} 4, & 0 \leq t < 5 \\ 1, & t \geq 5 \end{cases}$$

5. (4 points) Use a table of Laplace transforms to compute the transform of each function.

(a) $f(t) = 5 - e^{2t} + 6t$

(b) $f(t) = t^3 - te^t + e^{4t} \cos t$

6. (5 points) We have studied mass-spring systems that are described by equations of the form

$$mx'' + bx' + kx = F_0 \cos \gamma t.$$

For such a system, explain the difference between the *transient part* of the solution and the *steady-state part* of the solution.

7. (3 points) Explain what it means for a function to be of *exponential order*.