Math 240 - Final Exam A   Name _____

May 5, 2022   Score _____

Show all work to receive full credit. You must work individually. With the exception of problem 4, this test is due May 10. If your approach to any problem on the test requires a partial fraction decomposition, you may use technology to find your PFD. All integration must be done by hand, unless otherwise indicated.

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1. (10 points) Consider the equation $x^2 \dfrac{dy}{dx} = 4x^2 + 7xy + 2y^2$.

    (a) Argue that the equation has a unique solution through any point where $x \neq 0$.

    (b) Solve the equation.

2. (15 points) Use variation of parameters to solve $y'' + 6y' + 34y = e^{-3t}\cot(5t)$.

3. (15 points) Use Laplace transform methods to solve the following equation.

$$xy'' + y' + xy = 0, \quad y(0) = 2$$

Your final answer will involve a special function called a Bessel function. In fact, your answer should involve $J_0(x)$ which is defined in SageMath by `bessel_J(0,x)`. And incidentally, $J_0(0) = 1$.

4. (10 points. Due May 12.) Solve the following one-dimensional heat equation with Dirichlet boundary conditions. Rather than derive the solution method (as we did in class), use Theorem 1 on page 593.

$$2\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0, t) = u(1, t) = 0,$$

$$u(x, 0) = x^2$$