## Math 240 - Final Exam B

May 12, 2022
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Show all work to receive full credit. All integration must be done by hand.

1. (10 points) Argue that the equation has a unique solution through any point where $x \neq 0$. Then solve the equation.

$$
\frac{1}{x} \frac{d y}{d x}-\frac{2 y}{x^{2}}=x \cos x, \quad x>0
$$

2. (10 points) Solve the initial value problem.

$$
y^{\prime \prime}+3 y^{\prime}+2 y=\sin x ; \quad y(0)=0, \quad y^{\prime}(0)=0
$$

3. (10 points) State the recurrence relation that describes the coefficients of the power series solution centered at $x=0$.

$$
y^{\prime}+(x+2) y=0
$$

4. (10 points) Use Laplace transform techniques to solve the initial value problem. (See the note below for future use.)

$$
y^{\prime \prime}-2 y^{\prime}+5 y=-8 e^{-t} ; \quad y(0)=2, y^{\prime}(0)=12
$$

Note: The PFD of your $Y(s)$ should be $\frac{3 s+5}{s^{2}-2 s+5}-\frac{1}{s+1}$.

