

Math 240 - Quiz 10

April 20, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3.5 points) Find the partial fraction decomposition of $Y(s) = \frac{4}{(s-1)(s+1)}$. Then find the inverse Laplace transform of $Y(s)$.

$$Y(s) = \frac{A}{s-1} + \frac{B}{s+1} \quad \text{Cover up} \Rightarrow A = \frac{4}{2}, \quad B = \frac{4}{-2}$$

$$Y(s) = \frac{2}{s-1} - \frac{2}{s+1}$$

$$y(t) = 2e^t - 2e^{-t}$$

2. (3 points) Use Laplace transforms to solve $y' = y - 4e^{-t}$, $y(0) = 1$.

$$y' - y = -4e^{-t} \Rightarrow sY - y(0) - Y = \frac{-4}{s+1}$$

$$sY - 1 - Y = \frac{-4}{s+1} \Rightarrow (s-1)Y = 1 - \frac{4}{s+1}$$

$$Y(s) = \frac{1}{s-1} - \frac{4}{(s-1)(s+1)} \quad \leftarrow \text{Use \#1}$$

$$y(t) = e^t - 2e^t + 2e^{-t} \Rightarrow y(t) = 2e^{-t} - e^t$$

3. (3.5 points) Find the inverse Laplace transform of $Y(s) = \frac{s-3}{s^2+6s+10}$.

$$Y(s) = \frac{s-3}{(s+3)^2+1} = \frac{s+3}{(s+3)^2+1} - \frac{6}{(s+3)^2+1}$$

$$y(t) = e^{-3t} \cos t - 6e^{-3t} \sin t$$