Name _____ key _____

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Use convolution to determine the inverse transform of $Y(s) = \dfrac{2}{s(s-1)}$.

$F(s) = \dfrac{2}{s}, \quad G(s) = \dfrac{1}{s-1}$

$f(t) = 2, \quad g(t) = e^t$

$\displaystyle\int_0^t e^{\tau} \cdot 2 \, d\tau$

$= 2e^{\tau} \Big|_0^t = \boxed{2e^t - 2}$

$\mathcal{L}^{-1}\{Y(s)\}(t) = (g * f)(t)$

2. (3 points) Use the derivative-of-transform theorem to compute the Laplace transform of $f(t) = t^2 e^{5t}$. Use your table to check that your answer is correct.

$\mathcal{L}\{t^2 e^{5t}\}(s) = \dfrac{d^2}{ds^2}\left(\mathcal{L}\{e^{5t}\}(s)\right) = \dfrac{d^2}{ds^2}\left(\dfrac{1}{s-5}\right) = \boxed{\dfrac{2}{(s-5)^3}}$

$f(s) = (s-5)^{-1}$

$f'(s) = -1(s-5)^{-2}$

$f''(s) = 2(s-5)^{-3}$

3. (4 points) Use Laplace transforms to transform the 2nd-order equation for $x(t)$ into a 1st-order equation for $X(s)$. Do not solve.

$$tx'' - x' + tx = 0, \quad x(0) = 0$$

$\mathcal{L}\{tx''\} - \mathcal{L}\{x'\} + \mathcal{L}\{tx\} = 0$

$-\dfrac{d}{ds}\left(s^2 X - sx(0) - x(0)\right) - \left(sX - x(0)\right) - \dfrac{d}{ds}X = 0$

$\overset{0}{} \quad \overset{\text{CONST}}{} \quad \overset{0}{}$

$-s^2 X' - 2sX - sX - X' = 0$

$(s^2 + 1)X' + 3sX = 0 \qquad \text{OR} \qquad \boxed{X' = \dfrac{-3s}{s^2+1}X}$