Math 240 - Quiz 12

May 4, 2023

Name Key

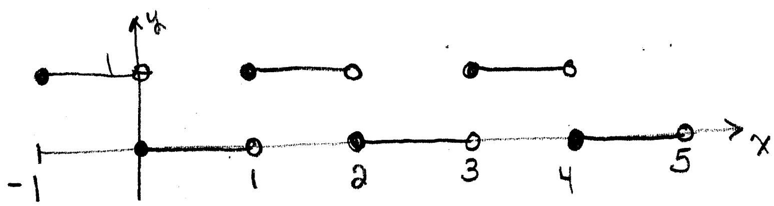
Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due May 9.

1. (10 points) Let $f(x)$ be the periodic extension (with period 2) of its portion defined on $[0,2)$ as shown below.

$$f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$$

(a) Roughly sketch the graph of 3 or 4 periods of f.



(b) Determine the Fourier series for f.

$$a_0 = \int_1^2 1\,dx =$$

$$a_n = \int_1^2 \cos n\pi x\,dx = \frac{1}{n\pi}\sin n\pi x\Big|_1^2 = 0$$

$$b_n = \int_1^2 \sin n\pi x\,dx = -\frac{1}{n\pi}\cos n\pi x\Big|_1^2 = -\frac{1}{n\pi}\left[\cos(2n\pi) - \cos(n\pi)\right]$$

$$= -\frac{1}{n\pi}\left[1-(-1)^n\right] = \frac{(-1)^n - 1}{n\pi}; \quad n = 1,2,3,\ldots$$

$$= \frac{-2}{\pi}, 0, \frac{-2}{3\pi}, 0, \ldots$$

$$\boxed{f(x) \sim \frac{1}{2} + \sum_{n=0}^{\infty} \frac{-2}{(2n+1)\pi}\sin\left((2n+1)\pi x\right)}$$

(c) Explain the difference between the Fourier series, the Fourier sine series, and the Fourier cosine series for f. (You don't need to compute them. Just explain.)

SEE BACK

Sine series $L=2$

$$b_n = \int_1^2 \sin\left(\frac{n\pi x}{2}\right)dx = -\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big|_1^2 = -\frac{2}{n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

Cosine series $L=2$

$$a_0 = \int_1^2 1\,dx = 1$$

$$a_n = \int_1^2 \cos\left(\frac{n\pi x}{2}\right)dx = \frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Big|_1^2 = \frac{-2}{n\pi}\sin\frac{n\pi}{2}$$

Suppose f is defined on $[0, L]$.

1) The Fourier cosine series of f is the Fourier series of the even extension of f to $[-L, L]$.

2) The Fourier sine series of f is the Fourier series of the odd extension of f to $[-L, L]$.

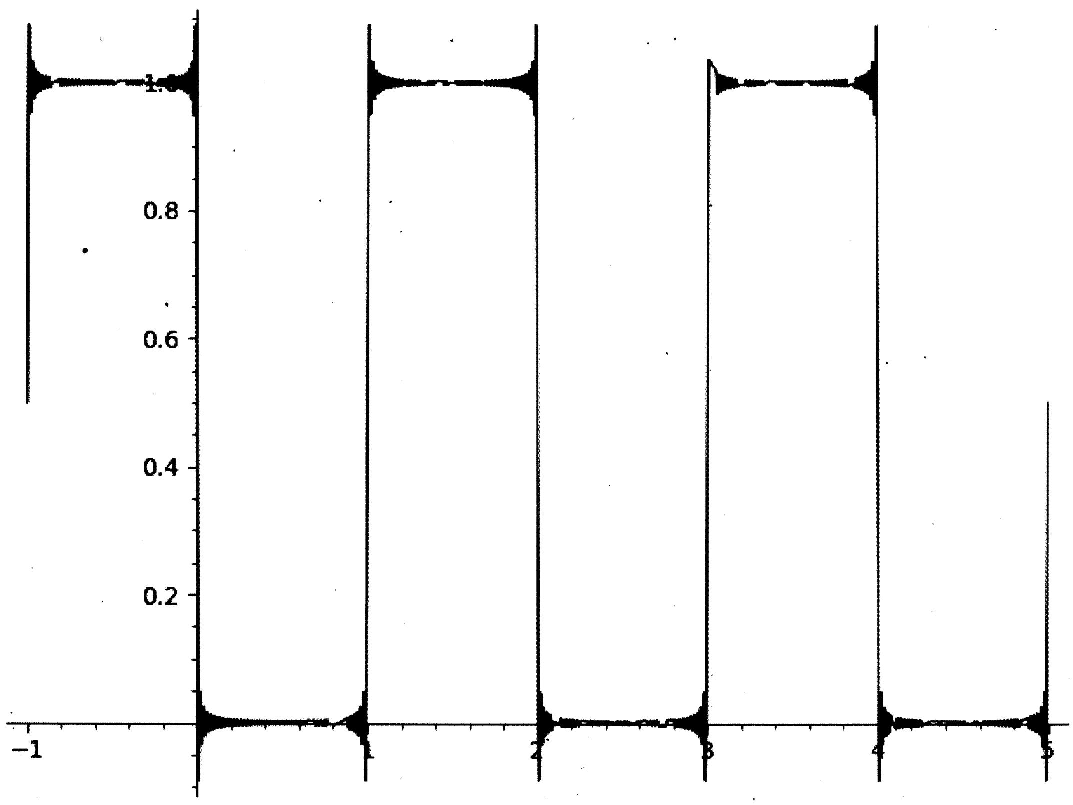
The function in the problem is neither even nor odd.

The graph of its Fourier series is attached (Page A).

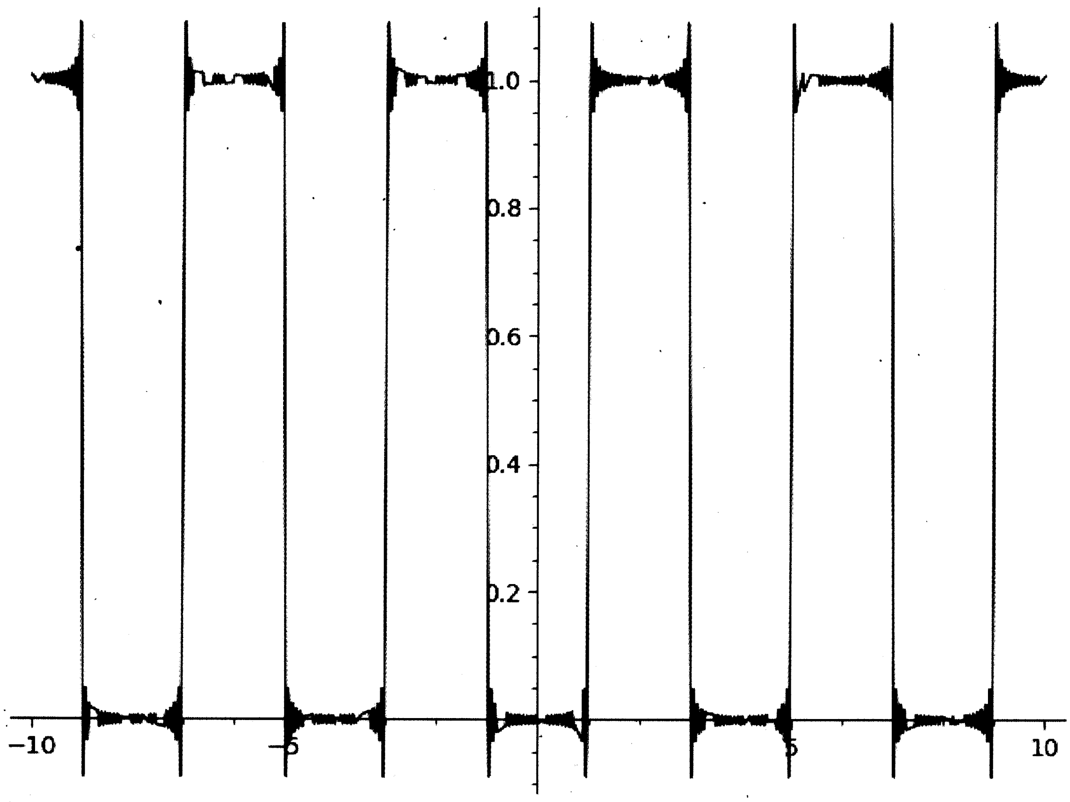
The Fourier series of its even extension to period 4 (its cosine series) is page B.

Sine series is page C.

Page A



Page B



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