Math 240 - Quiz 2
January 26, 2023

Name key

Score

Supply explanations if necessary.

1. (3 points) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{10}{x^2+1}, \quad y(0) = 0$$

$y(x) = \int \frac{10}{x^2+1}\,dx.$

$= 10\tan^{-1}x + C$

$\boxed{y(x) = 10\tan^{-1}x}$.

$y(0) = 0 = 10\tan^{-1}(0) + C$

$= 0 + C \Rightarrow C = 0$

2. (3 points) Suppose you are sketching the direction field for the differential equation

$$x^2\frac{dy}{dx} + 3xy^3 = 4.$$

(a) What is the slope of the solution curve passing through $(2,3)$?

$\frac{dy}{dx} = \frac{4-3xy^3}{x^2}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{4-3(2)(3)^3}{(2)^2} = \boxed{\frac{-158}{4} = -39.5}$

(b) Find a point through which you would not expect a solution curve to exist. Say why.

$f(x,y) = \frac{4-3xy^3}{x^2}$

f IS NOT CONTINUOUS WHEN $X=0$.

FROM OUR EXISTENCE THEOREM, I WOULD NOT EXPECT A SOL'N THROUGH ANY POINT WHERE $X=0$, e.g. $(0,0)$.

Turn over.

3. (4 points) Analyze each initial value problem and determine whether we could expect a unique solution, more than one solution, or no solution to exist through the given point.

(a) $\frac{dy}{dx} - x^2 y = \sin^3 x, \quad y(\pi) = 2$

$$\left.\begin{array}{l} f(x,y) = x^2 y + \sin^3 x \\ f_y(x,y) = x^2 \end{array}\right\}$$

THESE ARE CONTINUOUS EVERYWHERE IN $\mathbb{R}^2$.

THERE IS A UNIQUE SOLUTION THROUGH ANY POINT.

(b) $y\frac{dy}{dx} = e^x, \quad y(1) = 0$

$$f(x,y) = \frac{e^x}{y}$$

THIS IS NOT CONTINUOUS WHEN $y = 0$.

I DO NOT EXPECT A SOLUTION THROUGH $(1,0)$.