

# Math 240 - Quiz 3

February 2, 2023

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Solve:  $y dy = 4x(y^2 + 1)^{1/2} dx$ ,  $y(1) = 2$

$$\int \frac{y}{\sqrt{y^2+1}} dy = \int 4x dx$$

$$u = y^2 + 1$$

$$du = 2y dy$$

$$\frac{1}{2} du = y dy$$

$$\frac{1}{2} \int u^{-1/2} du = 2x^2 + C$$

$$u^{1/2} = 2x^2 + C$$

$$\sqrt{y^2+1} = 2x^2 + C$$

$$y(1) = 2 \Rightarrow \sqrt{5} = 2 + C$$

$$C = \sqrt{5} - 2$$

$$\boxed{\sqrt{y^2+1} = 2x^2 + \sqrt{5} - 2}$$

2. (5 points) Use our method for solving linear equations to solve the initial value problem.

$$\frac{dy}{dx} + 2xy = x, \quad y(0) = -3$$

$$\mu(x) = e^{\int 2x dx} = e^{x^2}$$

$$e^{x^2} y(x) = \int x e^{x^2} dx = \int \frac{1}{2} e^u du = \frac{1}{2} e^{x^2} + C$$

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$y(x) = \frac{1}{2} + C e^{-x^2}$$

$$y(0) = -3 \Rightarrow -3 = \frac{1}{2} + C$$
$$C = -\frac{7}{2}$$

$$\boxed{y(x) = \frac{1}{2} - \frac{7}{2} e^{-x^2}}$$

3. (1 point) Show that the equation in problem 2 is separable (as well as being linear).

$$\frac{dy}{dx} = x - 2xy = x(1 - 2y) \Rightarrow \frac{dy}{1-2y} = x dx$$

SEPARABLE