

Math 240 - Quiz 5
February 23, 2023

Name key Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Solve: $y'' - 3y' - 10y = 0$; $y(0) = 3$, $y'(0) = -4$

CHAR. EQN: $r^2 - 3r - 10 = 0$

$$(r-5)(r+2) = 0$$

$$r = 5, r = -2$$

$$y(x) = c_1 e^{5x} + c_2 e^{-2x}$$

$$y(0) = 3 \Rightarrow c_1 + c_2 = 3$$

$$y'(0) = -4 \Rightarrow 5c_1 - 2c_2 = -4$$

$$7c_1 = 2 \Rightarrow c_1 = \frac{2}{7}$$

$$\frac{2}{7} + c_2 = 3 \Rightarrow c_2 = \frac{19}{7}$$

$$y(x) = \frac{2}{7} e^{5x} + \frac{19}{7} e^{-2x}$$

2. (4 points) Solve: $y''' - 8y'' + 16y' = 0$

CHAR EQN: $r^3 - 8r^2 + 16r = 0$

$$r(r-4)(r-4) = 0$$

$$r = 0, r = 4, r = 4$$

$$y(x) = c_1 + c_2 e^{4x} + c_3 x e^{4x}$$

3. (1 points) It is easy to verify that $y_1(x) \equiv 1$ and $y_2(x) = \ln x$ are linearly independent solutions of $y'' + (y')^2 = 0$. It is also easy to verify that $y(x) = c_1 y_1(x) + c_2 y_2(x)$ is, in general, NOT a solution. Explain why the linear combination of solutions is not a solution.

THE EQUATION $y'' + (y')^2 = 0$ IS NOT A LINEAR EQUATION.

WE SHOULD NOT EXPECT A LINEAR COMBINATION OF SOLUTIONS
TO BE A SOLUTION.