Math 240 - Quiz 6

March 2, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Consider the following initial value problem:

$$y'' + 2y' - 15y = -32e^{-x},\ y(0) = 1, y'(0) = -29.$$

(a) Verify that $y_p(x) = 2e^{-x}$ is a particular solution of the equation.

$y_p' = -2e^{-x}$

$y_p'' = 2e^{-x}$

$2e^{-x} + 2(-2e^{-x}) - 15(2e^{-x})$

$= (2-4-30)e^{-x} = -32e^{-x}$ ✓

(b) Find the solution.

$r^2 + 2r - 15 = 0$

$(r+5)(r-3) = 0$

$r = -5,\ r = 3$

$y_h(x) = c_1 e^{-5x} + c_2 e^{3x}$

$\boxed{y(x) = 2e^{-x} + c_1 e^{-5x} + c_2 e^{3x}}$

$y(0) = 1 \Rightarrow 2 + c_1 + c_2 = 1$

$y'(0) = -29 \Rightarrow -2 - 5c_1 + 3c_2 = -29$

$c_1 + c_2 = -1$

$-5c_1 + 3c_2 = -27$

$8c_2 = -32$

$\boxed{c_2 = -4} \Rightarrow \boxed{c_1 = 3}$

2. (5 points) Solve the Cauchy-Euler equation $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + 3y = 0$.

$x = e^t,\ x > 0$

$\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 3y = 0$

$r^2 + 2r + 3 = 0$

$r^2 + 2r + 1 = -2$

$(r+1)^2 = -2$

$r = -1 \pm \sqrt{2}\,i$

$y(t) = c_1 e^{-t}\cos(\sqrt{2}\,t) + c_2 e^{-t}\sin(\sqrt{2}\,t)$

$\boxed{y(x) = \dfrac{c_1}{x}\cos(\sqrt{2}\ln x) + \dfrac{c_2}{x}\sin(\sqrt{2}\ln x)}$