Math 240 - Quiz 8

March 30, 2023

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Find the first five nonzero terms of the power series solution centered at $x = 0$.

$$(x-2)y' + y = 0$$

$$xy' - 2y' + y = 0$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n, \quad y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$0 = \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=1}^{\infty} 2n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

Replace n w/ $n+1$

$$= \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} 2(n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} 2(n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} \left[n a_n - 2(n+1) a_{n+1} + a_n \right] x^n$$

$$n a_n - 2(n+1) a_{n+1} + a_n = 0$$

$$a_{n+1} = \frac{1}{2} a_n \; ; \quad n = 0, 1, 2, 3, \ldots$$

a_0 = ARBITRARY

$a_1 = \frac{1}{2} a_0$

$a_2 = \frac{1}{4} a_0$

$a_3 = \frac{1}{8} a_0$

$a_4 = \frac{1}{16} a_0$

$a_5 = \frac{1}{32} a_0$

$\vdots$

$$a_{n+1} = \frac{1}{2^{n+1}} a_0$$

$$y(x) = a_0 \left(1 + \frac{1}{2} x + \frac{1}{4} x^2 + \frac{1}{8} x^3 + \frac{1}{16} x^4 + \cdots \right)$$

$$y(x) = a_0 \sum_{n=0}^{\infty} \left(\frac{x}{2} \right)^n$$

$$y(x) = \frac{a_0}{1 - \frac{x}{2}} = \frac{C}{2 - x}$$