<u>Math 240 - Test 2</u>

March 9, 2023

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary. Give explicit solutions when possible. All integration must be done by hand, unless otherwise specified.

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1. (8 points) Solve the initial value problem.

$$y'' + 25y = 0; \quad y(0) = 10,\, y'(0) = -10$$

2. (8 points) Solve $yy'' = (y')^2$, assuming y and y' are positive.

3. (4 points) Explain why the functions $y_1(x) = 1 + 2x$, $y_2(x) = 3 + x^2$, and $y_3(x) = 5 + 4x + x^2$ are linearly **dependent**.

4. (6 points) The general solution of a homogeneous, constant-coefficient, 2nd-order, linear differential equation is $y(x) = c_1 e^{3x} + c_2 x e^{3x}$. Find such an equation.

5. (10 points) Solve the initial value problem.

$$x^2 y'' - 2xy' - 10y = 0; \quad y(1) = 5,\ y'(1) = 8$$

6. (4 points) Suppose that the functions y_1 and y_2 are solutions of the equation $xy'' + (5 - x^2)y' + 2xy = x^2 + x$. Would you also expect $y(x) = y_1(x) + y_2(x)$ to be a solution? Explain your reasoning.

7. (8 points) A homogeneous, constant-coefficient, linear differential equation has the following characteristic equation:

$$r(r-1)^4(r^2 + 2r + 10)^2 = 0.$$

Find the general solution of the original differential equation.

8. (4 points) It is easy to verify that $y_1(x) = x^2$ and $y_2(x) = x^3$ are two **different**, linearly independent solutions of the initial value problem

$$x^2y'' - 4xy' + 6y = 0; \quad y(0) = 0,\, y'(0) = 0.$$

Explain why does this not contradict our existence/uniqueness theorem for linear equations?

9. (10 points) Consider the equation $(x^2+1)y'' - 2xy' + 2y = 0$.

(a) The functions $y_1(x) = x$ and $y_2(x) = 1 - x^2$ are solutions. Choose either one of them and verify that it is a solution.

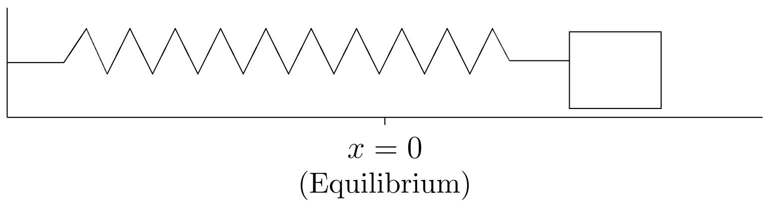
(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(-\infty, \infty)$.

(c) Now consider the nonhomogeneous equation $(x^2+1)y'' - 2xy' + 2y = 2x^3 + 6x$. Verify that $y_p(x) = x^3$ is a solution.

(d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $(x^2+1)y'' - 2xy' + 2y = 2x^3 + 6x$; $y(1) = 3$, $y'(1) = 3$.

10. (4 points) Give an example of constants m, b, and k for which the mass-spring system described by $mx'' + bx' + kx = 0$ would be critically damped. Describe the form of the solution in this case.

11. (14 points) A 1-kg mass is attached to a spring with spring constant $\frac{17}{4}$ N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 2 m to the left of equilibrium (compressing the spring) and released from rest. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



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The following problems make up the take-home portion of the test. These problems are due March 21, 2023. You must work on your own.

12. (15 points) Use undetermined coefficients to solve the following initial value problem.

$$y'' + 6y' + 5y = x + 8e^{-x}; \quad y(0) = 1,\, y'(0) = -1$$

13. (5 points) Consider the following equation:

$$y'' - 6y' + 10y = e^{3x}\cos x.$$

Solve the corresponding homogeneous equation. Then use your table to find the appropriate <u>form</u> of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.