Math 240 - Final Exam BNameMay 11, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) Use any method to find the general solution of y''' - 6y'' + 9y' = 0.

2. (8 points) Use any method to find the general solution of $y'' - y' - 6y = 5 + 2e^{-x}$.

3. (10 points) In this problem, you will find five (5) ordinary differential equations. Each equation has a specific name or can be described by a word or short phrase. For each equation, write that name or description, and then write a sentence describing a solution method. Be brief, but specific, when describing your solution method.

(a)
$$x^2y'' + 7xy' + 25y = 0$$

(b)
$$(x^2+1)\frac{dy}{dx} + 3xy = 6x$$

(c)
$$2xy\frac{dy}{dx} = 4x^2 + 3y^2$$

(d)
$$3y'' + xy' - 4y = 0$$

(e)
$$\frac{x}{y^2}\frac{dy}{dx} + \left(x - \frac{1}{y}\right) = 0$$

- 4. (16 points) Choose any two of the equations from problem 3 and solve each by using the solution method that you described above.
 - (a) First problem:

(b) Second problem:

5. (12 points) A 1-kg mass is attached to a spring with spring constant $\frac{17}{16}$ N/m. The damping constant for the system is $\frac{1}{2}$ N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the right at $\frac{3}{4}$ m/sec. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



Follow-up: Determine when the mass passes through equilibrium for the first time.