Math 240-Final Exam B Name
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Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) Use any method to find the general solution of $y^{\prime \prime \prime}-6 y^{\prime \prime}+9 y^{\prime}=0$.
2. (8 points) Use any method to find the general solution of $y^{\prime \prime}-y^{\prime}-6 y=5+2 e^{-x}$.
3. (10 points) In this problem, you will find five (5) ordinary differential equations. Each equation has a specific name or can be described by a word or short phrase. For each equation, write that name or description, and then write a sentence describing a solution method. Be brief, but specific, when describing your solution method.
(a) $x^{2} y^{\prime \prime}+7 x y^{\prime}+25 y=0$
(b) $\left(x^{2}+1\right) \frac{d y}{d x}+3 x y=6 x$
(c) $2 x y \frac{d y}{d x}=4 x^{2}+3 y^{2}$
(d) $3 y^{\prime \prime}+x y^{\prime}-4 y=0$
(e) $\frac{x}{y^{2}} \frac{d y}{d x}+\left(x-\frac{1}{y}\right)=0$
4. (16 points) Choose any two of the equations from problem 3 and solve each by using the solution method that you described above.
(a) First problem:
(b) Second problem:
5. (12 points) A $1-\mathrm{kg}$ mass is attached to a spring with spring constant $\frac{17}{16} \mathrm{~N} / \mathrm{m}$. The damping constant for the system is $\frac{1}{2} \mathrm{~N}-\mathrm{sec} / \mathrm{m}$. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the right at $\frac{3}{4} \mathrm{~m} / \mathrm{sec}$. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.


Follow-up: Determine when the mass passes through equilibrium for the first time.

