Math 240 - Assignment 2

January 25, 2024

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 1.

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1. Suppose the deer population $P(t)$ in a small forest satisfies the the equation
$$\frac{dP}{dt} = 0.0225P - 0.0003P^2.$$
Construct a slope field (use technology) and an approximate solution curve to answer (approximately) the following questions: If there are 25 deer at time $t = 0$ and t is measured in months, how long will it take for the number of deer to double? What is the limiting deer population?

2. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.

   (B) There is a unique solution.

   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.
$$\frac{dy}{dx} = \sqrt{x - y}, \quad y(2) = 2$$

3. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.

   (B) There is a unique solution.

   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.
$$y' - 3y^{2/3} = 0, \quad y(2) = 0$$

4. Use Euler's method (by hand) with $h = 0.1$ to approximate $y(0.3)$.
$$\frac{dy}{dx} = -\frac{2xy}{1 + x^2}, \quad y(0) = 1.$$
Follow-up: Use technology with $h = 0.01$ to approximate $y(0.3)$.

5. Solve the initial value problem: $\dfrac{dy}{dx} = 2xy^2 + 3x^2y^2, \quad y(1) = -1.$

6. An object at -2° C is placed into a room where the ambient temperature is 22° C. After 20 minutes the object has warmed to 10° C. Set up and solve the differential equation that gives the temperature of the object at time t ($t \geq 0$). What is the object's temperature after 40 minutes? (Use Newton's law of cooling.)

7. Solve the initial value problem: $xy' - 3y = x^3, \quad y(1) = 10.$

8. The equation in problem #1 is separable:

$$\frac{dP}{dt} = 0.0225P - 0.0003P^2, \quad P(0) = 25.$$

Solve the initial value problem. (If you use a partial fraction decomposition, feel free to use technology to find it.)