Math 240 - Assignment 3

Name _____

February 1, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 8.

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1. Solve the initial value problem: $xy' + 2y = 3x, \quad y(1) = 5.$

2. Solve the initial value problem: $y' = 2xy + 3x^2 e^{x^2}, \quad y(0) = 5.$

3. A tank initially contains 40 gal of pure water. A salt solution containing 3 lb of salt per gallon enters the tank at 2 gal/min and is uniformly mixed. The mixed solution leaves the tank at 3 gal/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up and solve the appropriate initial value problem to determine $A(t)$. How much salt is in the tank when the volume is 20 gal?

4. Suppose that a falling object is subject to two forces: the force of gravity and a force of air resistance proportional to the object's velocity. In such a case, Newton's 2nd law gives
$$m\frac{dv}{dt} = -mg - bv, \qquad v(0) = v_0,$$
where m is the mass of the object, v is the velocity at time t, g is the acceleration due to the force of gravity, and b is the constant of proportionality associated with the force of air resistance. Solve the initial value problem to find $v(t)$. Then compute $\lim_{t\to\infty} v(t)$ and explain what the result means.

5. Solve: $(2xy^2 + 3x^2)\,dx + (2x^2y + 4y^3)\,dy = 0$

6. Solve: $(x + \tan^{-1} y)\,dx + \dfrac{x+y}{1+y^2}\,dy = 0$

7. Consider the equation $(xy - 1)\,dx + (x^2 - xy)\,dy = 0$. Show that the equation is NOT exact. Then multiply both sides of the equation by $1/x$, and show that the new equation is exact. Solve the new equation.