# Math 240 - Assignment 4

Name _____

February 15, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 22.

---

1. Solve: $xy'' - y' = 3x^2$.
2. Solve the initial value problem: $y'' = y'e^y, \quad y(0) = 0,\ y'(0) = 1$.
3. Solve (twice) using both of our reduction of order strategies: $y'' = 1 + (y')^2$.
4. Consider the following ODE: $x^2y'' + 2xy' - 6y = 0$.
   (a) Verify that $y_1(x) = x^2$ and $y_2(x) = x^{-3}$ are solutions for $x > 0$.
   (b) Use the Wronskian to show that $y_1$ and $y_2$ are linearly independent on $(0, \infty)$.
   (c) Find the unique solution that satisfies $y(2) = 10$ and $y'(2) = 15$.
5. Consider the following ODE: $yy'' + (y')^2 = 0$.
   (a) Verify that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions.
   (b) Show that $y_1(x) + y_2(x)$ is not a solution.
   (c) Why should you not expect the sum of solutions to be a solution?
6. Find the general solution: $4y'' + 8y' + 3y = 0$.
7. Find the general solution: $9y'' - 12y' + 4y = 0$.
8. Find the general solution: $y^{(4)} - 2y''' + y'' = 0$.
9. Solve the following initial value problem.

$$2y'' - 2y' + y = 0; \quad y(0) = -1,\ y'(0) = 0$$

10. Find the general solution: $y^{(4)} - 6y''' + 3y'' + 8y' + 48y = 0$.
    (Hint: $r^4 - 6r^3 + 3r^2 + 8r + 48 = (r^2 - 8r + 16)(r^2 + 2r + 3)$)