Math 240 - Assignment 5

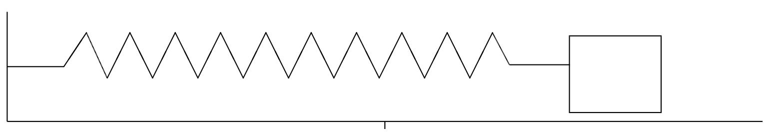
Name ____________________

February 22, 2024

Score __________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 29.

1. Find the general solution: $y^{(5)} + 2y^{(3)} = 0$
2. A homogeneous, constant-coefficient, linear differential equation has the following characteristic equation:
$$r(r-1)^4(r^2+2r+10)^2 = 0.$$
Find the general solution of the original differential equation.
3. Solve the Cauchy-Euler equation $x^2y'' + 7xy' + 25y = 0$.
4. Give an example of constants m, b, and k for which the mass-spring system described by $mx'' + bx' + kx = 0$ would be critically damped. Describe the form of the solution in this case.
5. A 1-kg mass is attached to a spring with spring constant $\frac{17}{4}$ N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 2 m to the left of equilibrium (compressing the spring) and released from rest. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



$x = 0$
(Equilibrium)

6. A 9-kg mass is attached to a spring with spring constant 37 N/m. The damping constant for the system is 6 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the left at 2 m/sec. (See the figure above.) Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift. When does the mass pass through equilibrium for the second time?
7. A 1-kg mass is attached to a spring with spring constant 16 N/m. The damping constant for the system is 10 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and pushed to the left at 12 m/sec. (See the figure above.) Find the equation of motion. Is the system underdamped, overdamped, or critically damped? How do you know?
8. Use undetermined coefficients to solve the following equation.
$$2y'' + 6y' - 20y = 60\sin 2x$$
9. Solve the initial value problem: $y'' - 5y' + 4y = 2e^{4x}$ $\quad y(0) = 1,\ y'(0) = -1$

10. Consider the following equation:

$$y'' - 10y' + 25y = 5x^2e^{5x}.$$

Solve the corresponding homogeneous equation. Then use your table to find the appropriate form of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.