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# MTH 240 - Assignment 5

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1)  $y^{(5)} + 2y^{(3)} = 0$

Char. eqn:  $r^5 + 2r^3 = 0$

$$r^3(r^2 + 2) = 0$$

$$r = 0, r = 0, r = 0, r = \pm 2i$$

$$y(x) = c_1 + c_2 x + c_3 x^3 + c_4 \cos 2x + c_5 \sin 2x$$

2) Char eqn:  $r(r-1)^4(r^2+2r+10)^2 = 0$

$$r = 0, r = 1, r = 1, r = 1, r = 1, r^2 + 2r + 10 = 0 \text{ (Twice)}$$

$$\uparrow$$
$$r^2 + 2r + 1 = -9$$

$$(r+1)^2 = -9$$

$$r = -1 \pm 3i \text{ (Twice)}$$

$$y(x) = c_1 + c_2 e^x + c_3 x e^x + c_4 x^2 e^x$$

$$+ c_5 x^3 e^x + c_6 e^{-x} \cos 3x + c_7 e^{-x} \sin 3x$$

$$+ c_8 x e^{-x} \cos 3x + c_9 x e^{-x} \sin 3x$$

3)  $x^2 y'' + 7xy' + 25y = 0$

$x = e^t \Rightarrow \frac{d^2 y}{dt^2} + 6 \frac{dy}{dt} + 25y = 0$

Char eqn:  $r^2 + 6r + 25 = 0$

$r^2 + 6r + 9 = -16$

$(r+3)^2 = -16$

$r = -3 \pm 4i$

$y(t) = c_1 e^{-3t} \cos 4t + c_2 e^{-3t} \sin 4t$

$x = e^t$

$y(x) = c_1 x^{-3} \cos(4 \ln x) + c_2 x^{-3} \sin(4 \ln x)$

4) CRIT DAMPED  $\Rightarrow b^2 - 4mk = 0$

$b = 4, m = 2, k = 2$

DE is  $2x'' + 4x' + 2x = 0$

Char eqn:  $2(r^2 + 2r + 1) = 0$

$2(r+1)^2 = 0 \Rightarrow r = -1$   
(Twice)

$x(t) = c_1 e^{-t} + c_2 t e^{-t}$

$$5) \quad x'' + x' + \frac{17}{4}x = 0, \quad x(0) = -2, \quad x'(0) = 0$$

$$r^2 + r + \frac{17}{4} = 0$$

$$r^2 + r + \frac{1}{4} = -\frac{16}{4}$$

$$\left(r + \frac{1}{2}\right)^2 = -\frac{16}{4}$$

$$r = -\frac{1}{2} \pm 2i$$

$$x(t) = c_1 e^{-\frac{1}{2}t} \cos 2t + c_2 e^{-\frac{1}{2}t} \sin 2t$$

$$x(0) = -2 \Rightarrow c_1 = -2$$

$$x'(t) = e^{-\frac{1}{2}t} \cos 2t + 4e^{-\frac{1}{2}t} \sin 2t - \frac{c_2}{2} e^{-\frac{1}{2}t} \sin 2t + 2c_2 e^{-\frac{1}{2}t} \cos 2t$$

$$x'(0) = 0 \Rightarrow 1 + 2c_2 = 0 \Rightarrow c_2 = -\frac{1}{2}$$

$$x(t) = -2e^{-\frac{1}{2}t} \cos 2t - \frac{1}{2}e^{-\frac{1}{2}t} \sin 2t$$

$$A = \sqrt{(-2)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$$

$$\tan \phi = \frac{-2}{-\frac{1}{2}} \quad \text{AND } \phi \text{ IN QUAD III.}$$

$$\phi = \tan^{-1}(4) + \pi$$

$$x(t) = \frac{\sqrt{17}}{2} e^{-\frac{t}{2}} \sin\left(2t + \pi + \tan^{-1}(4)\right)$$

$$6) \quad 9x'' + 6x' + 37x = 0; \quad x(0) = 1, \quad x'(0) = -2$$

$$9r^2 + 6r + 37 = 0$$

$$9r^2 + 6r + 1 = -36$$

$$(3r+1)^2 = -36$$

$$r = -\frac{1}{3} \pm \frac{6i}{3} = -\frac{1}{3} \pm 2i$$

$$X(t) = c_1 e^{-\frac{1}{3}t} \cos 2t + c_2 e^{-\frac{1}{3}t} \sin 2t$$

$$X(0) = 1 \Rightarrow c_1 = 1$$

$$X'(t) = -\frac{1}{3}e^{-\frac{1}{3}t} \cos 2t - 2e^{-\frac{1}{3}t} \sin 2t - \frac{c_2}{3}e^{-\frac{1}{3}t} \sin 2t + 2c_2 e^{-\frac{1}{3}t} \cos 2t$$

$$X'(0) = -2 \Rightarrow -\frac{1}{3} + 2c_2 = -2 \Rightarrow c_2 = -\frac{5}{6}$$

$$X(t) = e^{-\frac{1}{3}t} \cos 2t - \frac{5}{6} e^{-\frac{1}{3}t} \sin 2t$$

$$A = \sqrt{(1)^2 + \left(-\frac{5}{6}\right)^2} = \sqrt{\frac{61}{36}} = \frac{\sqrt{61}}{6}$$

$$\tan \phi = \frac{1}{-5/6} \quad \text{AND } \phi \text{ IN QUAD II}$$

$$\phi = \tan^{-1}\left(-\frac{6}{5}\right) + \pi$$

$$\Rightarrow X(t) = \frac{\sqrt{61}}{6} e^{-\frac{1}{3}t} \sin\left(2t + \pi + \tan^{-1}\left(-\frac{6}{5}\right)\right)$$

Follow-up...

$$X(t) = 0 \Leftrightarrow 2t + \pi + \tan^{-1}\left(-\frac{6}{5}\right) = k\pi$$

$$t = \frac{(k-1)\pi - \tan^{-1}\left(-\frac{6}{5}\right)}{2}$$

k=1 gives 1<sup>st</sup> TIME THROUGH EQ

$$k=2 \Rightarrow$$

$$t \approx 2.0088 \text{ sec}$$

(5)

$$7) \quad x'' + 10x' + 16x = 0; \quad x(0) = 1, \quad x'(0) = -12$$

$$r^2 + 10r + 16 = 0$$

$$(r+2)(r+8) = 0$$

$$r = -2, \quad r = -8$$

$$x(t) = c_1 e^{-2t} + c_2 e^{-8t}$$

$$x(0) = 1 \Rightarrow c_1 + c_2 = 1$$

$$x'(t) = -2c_1 e^{-2t} - 8c_2 e^{-8t}$$

$$x'(0) = -12 \Rightarrow -2c_1 - 8c_2 = -12$$

$$a \quad (c_1 + c_2 = 1)$$

$$-2c_1 - 8c_2 = -12$$

$$-6c_2 = -10$$

$$c_2 = \frac{5}{3}$$

$$c_1 = -\frac{2}{3}$$

$$x(t) = -\frac{2}{3} e^{-2t} + \frac{5}{3} e^{-8t}$$

**OVERDAMPED MOTION** - No oscillations.

8)  $y'' + 3y' - 10y = 30 \sin 2x$

Homo Eqn:

$r^2 + 3r - 10 = 0$

$(r+5)(r-2) = 0$

$r = -5, r = 2$

$y_c(x) = c_1 e^{-5x} + c_2 e^{2x}$

$y(x) = c_1 e^{-5x} + c_2 e^{2x} - \frac{45}{58} \cos 2x - \frac{105}{58} \sin 2x$

Non Homo Eqn:

$g(x) = 30 \sin 2x$

$y_p(x) = A \cos 2x + B \sin 2x$

$y_p'(x) = -2A \sin 2x + 2B \cos 2x$

$y_p''(x) = -4A \cos 2x - 4B \sin 2x$

$y_p'' + 3y_p' - 10y_p = -4A \cos 2x - 4B \sin 2x + 6B \cos 2x - 6A \sin 2x - 10A \cos 2x - 10B \sin 2x = 30 \sin 2x$

$-14A + 6B = 0$

$-6A - 14B = 30$

$A = \frac{\begin{vmatrix} 0 & 6 \\ 30 & -14 \end{vmatrix}}{\begin{vmatrix} -14 & 6 \\ -6 & -14 \end{vmatrix}} = \frac{-180}{232} = -\frac{45}{58}$

$B = \frac{\begin{vmatrix} -14 & 0 \\ -6 & 30 \end{vmatrix}}{\begin{vmatrix} -14 & 6 \\ -6 & -14 \end{vmatrix}} = \frac{-420}{232} = -\frac{105}{58}$

$y_p(x) = -\frac{45}{58} \cos 2x - \frac{105}{58} \sin 2x$

9)

THESE PROBLEMS ARE ALSO ON

10)

ASSIGNMENT 6. SEE KEY

FOR ASSIGNMENT 6.