Math 240 - Assignment 7

March 21, 2024

Name ______________

Score __________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due March 28.

1. Find the first eight nonzero terms of the power series solution centered at $x = 0$.

   $$(x-2)y' + y = 0$$

   Based on the pattern in your solution, write the power series solution as an infinite sum in summation notation.

2. Find a power series solution. Then refer to the Common Infinite Series handout to find the interval of convergence and a familiar expression for your solution.

   $$(x-10)y' + y = 0$$

3. State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

   $$(x^2-3)y'' + 2xy' = 0$$

4. State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

   $$5y'' - 2xy' + 10y = 0$$

5. State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

   $$y'' - x^2y' - 3xy = 0$$

6. Consider the following first-order linear equation.

   $$x^3y' = 2y$$

   (a) Find a power series solution centered at $x = 0$.

   (b) Use the techniques of section 1.5 to find the general solution.

   (c) Compare your solutions. Why did the power series approach fail to provide the general solution?

7. For each equation below, consider a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

   Determine the minimum radius of convergence that is guaranteed by the theorem we discussed in class.

   (a) $(x+4)y'' + 7xy' - 5(x+2)y = 0$

   (b) $(x^2+1)y'' + 8y' + (x^2-1)y = 0$