Math 240 - Assignment 9

April 4, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 11.

1. Use the definition of the Laplace transform to find the transform of f.

$$f(t) = \begin{cases} 4, & 0 \le t < 8 \\ 2, & t \ge 8 \end{cases}$$

2. Use the definition of the Laplace transform to find the transform of f.

$$f(t) = \begin{cases} 1 - t, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

3. Find the inverse Laplace transform of each function. Show how you got your answer.

   (a) $F(s) = \dfrac{3s + 1}{s^2 + 4}$

   (b) $F(s) = 5s^{-1}e^{-3s}$

   (c) $F(s) = \dfrac{3}{s} - \dfrac{2}{s^4} - \dfrac{8}{6 - s}$

4. Let $F(s) = \dfrac{3}{s(s + 5)}$. Find the inverse Laplace transform by first computing the partial fraction decomposition (by hand).

5. Find the inverse Laplace transform of $F(s)$. You may use technology to compute the partial fraction decomposition.

$$F(s) = \frac{s + 1}{s^2(s + 2)^3}$$

6. Use Laplace transform techniques to solve the initial value problem. You may use technology to compute any required partial fraction decompositions.

$$y'' + 16y = \sin t, \quad y(0) = 0, \ y'(0) = 1$$

7. Use Laplace transform techniques to solve the initial value problem. You may use technology to compute any required partial fraction decompositions.

$$y'' - 6y' + 9y = t^2 e^{3t}, \quad y(0) = 2, \ y'(0) = 6$$

8. Use Laplace transform techniques to solve. You may use technology to compute any required partial fraction decompositions.

$$y'' + 4y' + 4y = t^3 e^{-2t}; \quad y(0) = 5, \ y'(0) = -10$$