Math 240 - Final Exam B

Name _____

May 9, 2024

Score _____

Show all work to receive full credit. Supply explanations where necessary.

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1. (8 points) Find the general solution of $(x^2+1)\dfrac{dy}{dx} + 3xy = 6x$.

2. (10 points) Find the general solution: $y'' - 5y' + 4y = 2e^{4x}$

3. (10 points) **Do any two of the following three problems** for five (5) points each. If you attempt all three, cross out the one you do not wanted graded.

   (a) Find the general solution of $y''' - 4y'' + 4y' = 0$.

   (b) Solve: $(6xy - y^3)\,dx + (4y + 3x^2 - 3xy^2)\,dy = 0$

   (c) Find the inverse Laplace transform of $F(s) = \dfrac{s^2 + 1}{s(s-1)(s-2)}$.

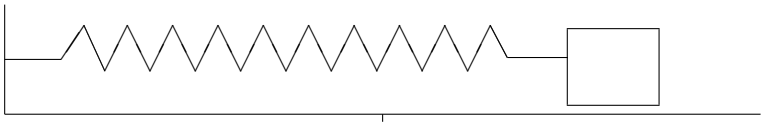
4. (10 points) State the complete recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

$$(x^2 - 3)y'' + 2xy' = 0$$

5. (10 points) Use Laplace transform techniques to solve.

$$y'' + 4y' + 4y = t^3 e^{-2t}; \quad y(0) = 5,\ y'(0) = -10$$

6. (12 points) A 4-kg mass is attached to a spring with spring constant $\frac{257}{16}$ N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 2 m to the right of equilibrium (stretching the spring) and released from rest. Find the equation of motion. Write your final answer in terms of a single sine or cosine with a phase shift.



$x = 0$
(Equilibrium)