

Math 240 - Assignment 1

January 23, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due January 30.

1. Show (by substitution) that both $y_1(x) = ae^{-5x}$ and $y_2(x) = bxe^{-5x}$ are solutions of $y'' + 10y' + 25y = 0$ for any constants a and b .
2. Classify the differential equation by saying whether it is ordinary or partial, linear or nonlinear. Also give its order and name the dependent and independent variables. Finally, show (by substitution) that $y = \frac{1}{x} - \ln x$ is a solution.

$$x^2y'' + xy' - y = \ln x$$

3. Is the following ordinary differential equation linear or nonlinear? Explain how you know. Then verify (by substitution) that $y = \ln(x + C)$ is a solution for any constant C . Finally, determine the constant C so that $y(0) = 0$.

$$e^y y' = 1$$

4. Solve the initial value problem: $\frac{dy}{dx} = xe^{-2x}$, $y(0) = 3$.

5. Write a differential equation that models the problem situation:

In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain disease is proportional to the product of the number who have the disease and the number who do not.

6. Read the problem situation below. Write a differential equation having $y = g(x)$ as one of its solutions.

The line tangent to the graph of g at (x, y) passes through the point $(x/2, 0)$.

7. Solve the initial value problem: $\frac{dy}{dx} = x\sqrt{x^2 + 9}$, $y(-4) = 0$.

8. An object is moving with constant acceleration $x''(t) = -20$. (Don't worry about units.) Find the position function $x(t)$ if $x(0) = 5$ and $x'(0) = -15$.