

# Math 240 - Assignment 3

February 6, 2025

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 13.

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1. Solve the initial value problem:  $xy' + 2y = 3x$ ,  $y(1) = 5$ .
2. Solve the initial value problem:  $y' = 2xy + 3x^2e^{x^2}$ ,  $y(0) = 5$ .
3. A tank initially contains 40 gal of pure water. A salt solution containing 3 lb of salt per gallon enters the tank at 2 gal/min and is uniformly mixed. The mixed solution leaves the tank at 3 gal/min. Let  $A(t)$  denote the amount of salt in the tank after  $t$  minutes. Set up and solve the appropriate initial value problem to determine  $A(t)$ . How much salt is in the tank when the volume is 20 gal?

4. Suppose that a falling object is subject to two forces: the force of gravity and a force of air resistance proportional to the object's velocity. In such a case, Newton's 2nd law gives

$$m \frac{dv}{dt} = -mg - bv, \quad v(0) = v_0,$$

where  $m$  is the mass of the object,  $v$  is the velocity at time  $t$ ,  $g$  is the acceleration due to the force of gravity, and  $b$  is the constant of proportionality associated with the force of air resistance. Solve the initial value problem to find  $v(t)$ . Then compute  $\lim_{t \rightarrow \infty} v(t)$  and explain what the result means.

5. Solve:  $(2xy^2 + 3x^2) dx + (2x^2y + 4y^3) dy = 0$
6. Solve:  $(x + \tan^{-1} y) dx + \frac{x + y}{1 + y^2} dy = 0$
7. Consider the equation  $(xy - 1) dx + (x^2 - xy) dy = 0$ . Show that the equation is NOT exact. Then multiply both sides of the equation by  $1/x$ , and show that the new equation is exact. Solve the new equation.