Math 240 - Assignment 1

January 22, 2026

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due January 29.

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1. Show (by substitution) that $y(x) = Cx^4e^{-2x}$ is a solution of the initial value problem

$$xy'' + (2x-3)y' + 2y = 0; \quad y(0) = 0,\, y'(0) = 0$$

for any constant C.

2. Let $y_1(t) = e^t$ and $y_2(t) = e^t \ln t$. Show (by substitution) that both y_1 and y_2 are solutions of

$$ty'' + (1-2t)y' + (t-1)y = 0.$$

Then show (by substitution) that $y(t) = \alpha y_1(t) + \beta y_2(t)$ is also solution for any constants α and β.

3. The following differential equation can be solved by straight-forward antidifferentiation. Find the general solution.

$$(1+x^2)y' = \tan^{-1} x$$

4. The equation below is the *Black-Scholes-Merton equation*, which describes the price evolution of financial derivatives. (Black and Scholes received the 1997 Nobel Prize in Economics.) Classify the differential equation by saying whether it is ordinary or partial, linear or nonlinear. Also give its order and name the dependent and independent variables.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV - rS\frac{\partial V}{\partial S}$$

5. Consider the equation $y' = 2xy + 1$. Is this differential equation linear or nonlinear? Explain how you know. Then verify (by substitution) that $y(x) = e^{x^2}\int_0^x e^{-t^2}\,dt$ is a solution.

6. For which values of m is $y(x) = e^{mx}$ a solution of $y'' - 5y' + 4y = 0$?

7. For a simple pendulum of fixed length ℓ, the acceleration of the pendulum bob is proportional to $\sin\theta$, where θ is the displacement angle from equilibrium. The acceleration is d^2s/dt^2, where $s = \ell\theta$. Write the differential equation for θ.

Turn over.

8. The graph of a function $y(x)$ rises in the first quadrant from the point $(0,0)$ to the point (x,y). The area under the graph is one third of the area of the rectangle with opposite vertices at $(0,0)$ and (x,y).

   (a) Briefly explain how the problem situation gives rise to the following equation:

   $$\int_0^x y(t)\,dt = \frac{1}{3}xy.$$

   (b) By differentiating both sides of the equation, show that the following differential equation describes the function $y(x)$.

   $$y = \frac{1}{3}xy' + \frac{1}{3}y.$$

9. Read the problem situation below. Write a differential equation having $y = g(x)$ as one of its solutions.

   The line normal to the graph of g at (x,y) passes through the point $(x/3, 1)$.

10. Solve the initial value problem: $\quad x(x^2-4)\dfrac{dy}{dx} = 1, \quad y(1) = 0.$