Math 240 - Assignment 2

January 29, 2026

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 5.

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1. Find the general solution: $\dfrac{dy}{dx} = 3x^2(1+y^2)$

2. Solve the initial value problem: $\dfrac{dy}{dx} = 8x^3e^{-2y}, \quad y(1) = 0.$

3. Solve the initial value problem: $\sqrt{y}\,dx + (1+x)\,dy = 0, \quad y(0) = 1.$

4. For $x \geq 0$, consider the initial value problem $dy/dx = y^{1/3}, \quad y(0) = 0.$
   (a) Use separation of variables to find the general solution of the equation. Then find the particular solution of the IVP that follows from your general solution.
   (b) Find a singular solution.

5. Based on our existence/uniqueness theorem, explain why you should not have expected a unique solution for the IVP in problem 4.

6. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.

   (B) There is a unique solution.

   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.

$$(y')^2 - xy' + y = 0, \quad y(2) = 1$$

Turn over.

7. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.
   (B) There is a unique solution.
   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.

$$\frac{dy}{dx} = y(y-1)(y-2), \quad y(0) = 3$$

8. Analyze the initial value problem to determine which one of these applies.

   (A) A solution exists, but it is not guaranteed to be unique.
   (B) There is a unique solution.
   (C) A solution is not guaranteed to exist.

   Be sure to show work or explain.

$$\frac{dy}{dx} = (x-3)(y+1)^{2/3}, \quad y(0) = -1$$

9. Suppose the population $P(t)$ (in thousands) of a certain species at time t satisfies the equation

$$\frac{dP}{dt} = 3P - 2P^2.$$

   Construct a slope field (use technology) to answer the following questions.

   (a) If the initial population is 2000, will the population increase or decrease? Quickly or slowly?
   (b) If the initial population is 2000, what is the limiting population?
   (c) If the initial population is 500, what is the limiting population?
   (d) Can a population of 3000 ever decline to 500?
   (e) If the initial population is 2000, what will happen to the population?
   (f) What will happen to the population if the initial population is exactly 1500?

10. Use Euler's method (by hand) with $h = 0.1$ to approximate $y(1.3)$.

$$\frac{dy}{dx} = y(2-y), \quad y(0) = 3.$$

    Follow-up: Use technology with $h = 0.01$ to approximate $y(1.3)$.

11. Use Euler's method (by hand) with $h = 0.5$ to approximate $y(1)$.

$$\frac{dy}{dx} = x + y, \quad y(0) = 1.$$

12. A cake at 300° F is removed from the oven and placed into a room where the ambient temperature is 70° F. After 3 minutes the cake has cooled to 200° F. Set up and solve the differential equation that gives the temperature of the cake at time t ($t \geq 0$). When will the cake reach 80° F? (Use Newton's law of cooling.)