Math 240 - Assignment 4

Name _____

February 19, 2026

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 26.

1. Solve the initial value problem: $y'' = y'e^y$, $y(0) = 0$, $y'(0) = 1$.

2. It is easy to verify (don't bother) that $y_1(x) = x^2$ and $y_2(x) = x^3$ are two **different**, linearly independent solutions of the initial value problem

$$x^2 y'' - 4xy' + 6y = 0; \quad y(0) = 0, \ y'(0) = 0.$$

Explain why does this not contradict our existence/uniqueness theorem for linear equations?

3. Explain why the functions $y_1(x) = x^2 + 1$, $y_2(x) = x^2 + 3x$, and $y_3(x) = 1 - 3x$ are linearly **dependent**. Do not use the Wronskian.

4. Consider the equation $(x - 1)y'' - xy' + y = 0$.

   (a) Verify that $y_1(x) = x$ and $y_2(x) = e^x$ are solutions.

   (b) Use the Wronskian to show that y_1 and y_2 are linearly independent on any interval not containing $x = 1$.

   (c) Now consider the nonhomogeneous equation $(x - 1)y'' - xy' + y = 2x - x^2$. Verify that $y_p(x) = 2 + x^2$ is a solution.

   (d) Use what you've learned in parts (a), (b), and (c) to find the solution of the IVP $(x - 1)y'' - xy' + y = 2x^2 - 4x$; $y(0) = 3$, $y'(0) = 9$.

   (e) Is your solution in part (d) unique? Explain.

5. Consider the following ODE: $yy'' + (y')^2 = 0$.

   (a) Verify that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions.

   (b) Show that $y_1(x) + y_2(x)$ is not a solution.

   (c) Why should you not expect the sum of solutions to be a solution?

6. Find the general solution: $6y'' - 7y' - 3y = 0$.

7. Find the general solution: $9y'' - 12y' + 4y = 0$.

8. Solve the following initial value problem.

$$y''' - 2y'' - y' + 2y = 0; \quad y(0) = 2, \ y'(0) = 3, \ y''(0) = 5$$

9. Find the general solution: $y''' - y'' + 7y' = 0$.

10. Solve the following initial value problem.

$$y'' - 2y' + 2y = 0; \quad y(\pi) = e^\pi, \ y'(\pi) = 0$$