Math 240 - Assignment 8

Name _____

April 2, 2026

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 9.

---

1. A 3-kg mass is attached to a spring with spring constant 5 N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 2 m to the **left** of equilibrium (compressing the spring) and pushed to the **right** at 1 m/sec. At the moment the mass is pushed, the periodic external force $F(t) = 5\cos 4t$ is applied.

$x = 0$
(Equilibrium)

   (a) Set up the initial value problem that describes the motion of the mass.

   (b) Use SageMath (or some other CAS) to solve the initial value problem. (If you need help with the SageMath syntax, see the posted lecture notes for section 2.6.)

   (c) Use SageMath (or some other CAS) to graph your solution for $0 \le t \le 32$. Attach a copy of the graph.

   (d) On your graph, indicate where the transient part of the solution is dominant and where the steady-state part is dominant.

   (e) Compute the gain factor for this system.

   (f) Compute the resonance frequency for this system.

2. For each equation below, consider a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

   Determine the minimum radius of convergence that is guaranteed by the theorem we discussed in class.

   (a) $(x-2)y'' - xy' + 9(x+2)y = 0$

   (b) $(x^2 - x)y'' - y' + 5(x+3)y = 0$

   (c) $(x^2 + 8)y'' + 2x^2y' + 6y = 0$

3. State the complete recurrence relation that describes the coefficients of the power series solution centered at $x = 0$. Also determine the minimum radius of convergence that is guaranteed by the theorem we discussed in class.

$$y'' - (1+x)y = 0$$

4. Consider the linear, 2nd-order equation $(1 - x^2)y'' - 6xy' - 4y = 0$. Determine the complete recurrence relation for the power series solution (centered at $x = 0$). Then write the first three terms of each of the two linearly independent solutions you obtain from your recurrence relations. Also determine the minimum radius of convergence that is guaranteed by the theorem we discussed in class.