

# Math 240 - Assignment 9

April 9, 2026

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 16.

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1. Consider the Cauchy-Euler equation  $x^2y'' + 8xy' + 10y = 0$ .

(a) Show that  $x = 0$  is a regular singular point.

(b) Assume  $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$  is a solution for some nonzero real number  $s$ . Differentiate, substitute, and equate coefficients as per our usual approach. (Because the series is not technically a power series, it is NOT appropriate to adjust the index when differentiating term by term.)

(c) Use  $a_0$  as an arbitrary constant and assume  $a_0 \neq 0$ . What does your recurrence relation tell you when  $n = 0$ ? (You should get what is called an *indicial equation*. That will give you two possible values for  $s$ . What are they?)

(d) One at a time, substitute your  $s$ -values into your recurrence relation from part (b) and solve for the coefficients  $a_n$ .

(e) Each  $s$ -value and the corresponding  $a_n$ 's gives you a solution for the original equation. What is the general solution?

(f) Use the techniques of chapter 2 to solve the equation, and then compare your results.

2. Use the definition of the Laplace transform to find the transform of  $f$ .

$$f(t) = \begin{cases} 2, & 0 \leq t < 5 \\ 3, & t \geq 5 \end{cases}$$

3. Use the definition of the Laplace transform to find the transform of  $f$ .

$$f(t) = \begin{cases} t + 1, & 0 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

4. Explain why the function  $f(t) = \sin t$ ,  $t \geq 0$  is of exponential order.

5. Explain why the function  $f(t) = e^{t^2}$ ,  $t \geq 0$  is not of exponential order.

6. Find the inverse Laplace transform of each function. Show how you got your answer.

(a)  $F(s) = \frac{3s + 1}{s^2 + 4}$

(b)  $F(s) = 5s^{-1}e^{-3s}$

(c)  $F(s) = \frac{3}{s} - \frac{2}{s^4} - \frac{8}{6 - s}$

*Turn over.*

7. Let  $F(s) = \frac{3}{s(s+5)}$ . Find the inverse Laplace transform by first computing the partial fraction decomposition (by hand).
8. Find the inverse Laplace transform of  $F(s)$ . You may use technology to compute the partial fraction decomposition.

$$F(s) = \frac{s+1}{s^2(s+2)^3}$$