

Math 240 - Test 2

March 12, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. All integration must be done by hand, unless otherwise specified.

1. (8 points) Find the general solution: $y^{(4)} + 4y''' + 5y'' = 0$

2. (8 points) Solve the following initial value problem.

$$2y'' + 10y' - 28y = 0; \quad y(0) = 1, \quad y'(0) = 29$$

3. (12 points) Consider the equation $xy'' + y' = 0$, $x > 0$. It is easy to verify (don't bother) that $y_1(x) = 1$ and $y_2(x) = \ln x$ are solutions.

(a) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(b) Now consider the nonhomogeneous equation $xy'' + y' = 1 + 8x$, $x > 0$. Verify that $y_p(x) = x + 2x^2$ is a solution.

(c) Use the information above to find the solution of the IVP $xy'' + y' = 3 + 24x$; $y(1) = 12$, $y'(1) = 20$.

(d) Is your solution in part (d) unique? Explain.

4. (8 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

(a) $5x'' + 12x' + 10x = 0$

(b) $x(t) = 6te^{-t/4} - 3e^{-t/4}$

(c) $3x'' + 3x' + \frac{1}{2}x = 0$

(d) $x(t) = \frac{\sqrt{17}}{4} \cos(\sqrt{3}t + 1)$

5. (14 points) Find the general solution: $3y'' - 11y' - 4y = 8e^{4x}$

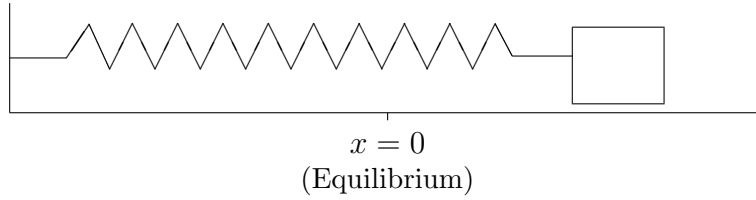
6. (8 points) Assume $x > 0$ and find the general solution: $x^2y'' + 7xy' + 9y = 0$

7. (8 points) Consider the following equation:

$$y'' - y = (x^2 + 1)e^x + \cos x.$$

Solve the corresponding homogeneous equation. Then use your table to find the appropriate form of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

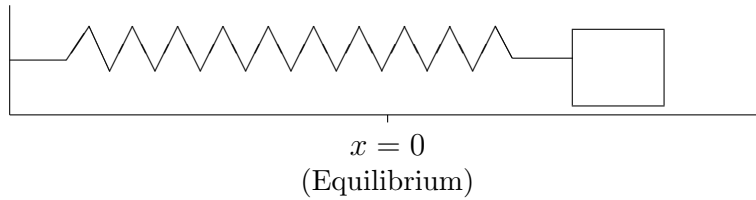
8. (16 points) A 2-kg mass is attached to a spring with spring constant 6 N/m. The damping constant for the system is 4 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and released from rest. Find the equation of motion. If applicable, write your solution in terms of a single sine or cosine with a phase shift.



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The following problems make up the take-home portion of the test. These problems are due March 24, 2026. You must work on your own. Evaluate any integrals by hand, showing all work.

9. (6 points) A $1/2$ -kg mass is attached to a spring with spring constant 5 N/m. The damping constant for the system is 1 N-sec/m. The mass is moved 2 m to the left of equilibrium (compressing the spring) and released from rest. Find the equation of motion. Write your final result in terms of a single trig function with a phase shift. When does the mass pass through equilibrium for the 2nd time?



10. (12 points) Solve the following initial value problem involving a nonhomogeneous Cauchy-Euler equation. (Use variation of parameters for the nonhomogeneous equation. For the corresponding homogeneous equation, if you do not use the approach we studied in class, you must show all details of your solution method.)

$$x^2y'' - xy' + 2y = x; \quad y(1) = 1, y'(1) = 1$$