Math 240 - Final Exam B

Name _____

May 14, 2026

Score _____

Show all work to receive full credit. Supply explanations where necessary.

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1. (10 points) A 100-kg object is launched downward with an initial speed of 10 m/s. With both gravity and air resistance acting on the object, Newton's 2nd law states that the velocity of the object, $v(t)$, in meters per second, satifies the equation

$$100\frac{dv}{dt} = -980 - 5v.$$

   (a) Find a formula for the object's velocity at time t.

   (b) When will the object's speed reach 120 m/s?

   (c) Determine the object's terminal velocity by computing $\lim_{t\to\infty} v(t)$.

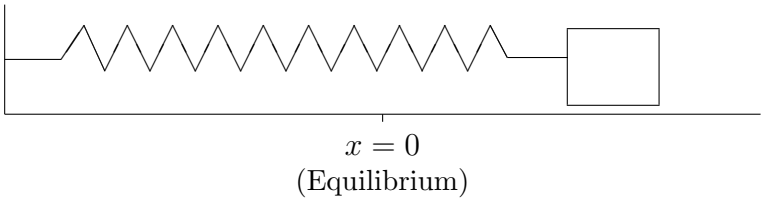
2. (10 points) Solve the initial value problem: $(x^2+1)\dfrac{dy}{dx} = 24x^3 - 4xy; \quad y(0) = 8$

3. (10 points) Find the general solution of each equation.

$$y''' + 3y'' - y' - 3y = 0$$

4. (10 points) Use variation of parameters to solve $y'' - 2y' + y = \dfrac{e^x}{x^2}$.

5. (10 points) A 2-kg mass is attached to a spring with spring constant 24 N/m. The damping constant for the system is 8 N-sec/m. The mass starts at the equilibrium position with an initial speed of 2 m/sec to the left. Set up and solve the initial value problem that describes the displacement of the mass from equilibrium.



6. (10 points) The equation

$$y'' - 2xy' + 6y = 0$$

is an example of Hermite's differential equation.

(a) Find the complete recurrence relation for a power series solution centered at $x = 0$.

(b) One of the two linearly independent solutions is a polynomial of degree 3. Find it.

(c) Find the first four nonzero terms of the second solution.

7. (10 points) Use Laplace transforms to solve the initial value problem.

$$x''' + x'' - 6x' = e^{4t}; \quad x(0) = 0,\ x'(0) = 1,\ x''(0) = 1$$