Solving by Completing the Square

To solve a quadratic equation by completing the square[1]:

1. Rewrite the equation so that all terms with variables appear on the left hand side of the equation and all constants appear on the right.
2. Combine like terms and arrange terms in order of descending exponents.
3. Divide both sides by the coefficient of the 2nd degree term, if that coefficient is not 1.
4. Take half the coefficient of the 1st degree term, square it, and add it to both sides of the equation.
5. Express the left hand side of the equation as the square of a binomial.
6. Undo the square by taking square roots. Remember that we want both square roots.
7. Solve by isolating the variable.
8. Simplify.

Example

Solve by completing the square: $3x^2 + 2x - 7 = 0$.

1. $3x^2 + 2x = 7$
2. Nothing to do in this step. $3x^2 + 2x = 7$
3. Divide by 3. $x^2 + \frac{2}{3}x = \frac{7}{3}$
4. Compute $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{2}{6}$ and add the square of this number to both sides. $x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{7}{3} + \frac{1}{9}$
5. $\left(x + \frac{1}{3}\right)^2 = \frac{22}{9}$
6. $x + \frac{1}{3} = \pm\sqrt{\frac{22}{9}}$
7. $x = \pm\sqrt{\frac{22}{9}} - \frac{1}{3}$
8. $x = \frac{-1 \pm \sqrt{22}}{3}$

[1] In general, completing the square involves rewriting a quadratic expression so that it contains a perfect square. For instance, to complete the square for the function $f(x) = x^2 + 2x + 4$ means to write it as $f(x) = (x+1)^2 + 3$.