Zeros of a Quartic Polynomial

The solution of the general 4th degree polynomial equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ was discovered around 1540 by Ludovico Ferrari, a colleague of Gerolamo Cardano. The technique depended upon reducing the quartic equation to a related cubic equation.

To solve the general quartic equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0, \qquad a \neq 0,$$

first let

$$a_1 = \frac{b}{a} \qquad a_2 = \frac{c}{a} \qquad a_3 = \frac{d}{a} \qquad a_4 = \frac{e}{a}$$

and let r be any real solution of the equation

$$y^3 - a_2 y^2 + (a_1 a_3 - 4a_4)y + (4a_2 a_4 - a_3^2 - a_1^2 a_4) = 0.$$

The four solutions of the quartic equation are now given by the four solutions of

$$z^2 + \frac{1}{2}\left(a_1 \pm \sqrt{a_1^2 - 4a_2 + 4r}\right) z + \frac{1}{2}\left(r \mp \sqrt{r^2 - 4a_4}\right) = 0.$$