



$$\text{Semiperimeter} = s = \frac{1}{2}(a + b + c)$$

### Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Law of Tangents

$$\frac{a + b}{a - b} = \frac{\tan \frac{1}{2}(\alpha + \beta)}{\tan \frac{1}{2}(\alpha - \beta)}$$

$$\frac{a + c}{a - c} = \frac{\tan \frac{1}{2}(\alpha + \gamma)}{\tan \frac{1}{2}(\alpha - \gamma)}$$

$$\frac{b + c}{b - c} = \frac{\tan \frac{1}{2}(\beta + \gamma)}{\tan \frac{1}{2}(\beta - \gamma)}$$

### Mollweide's Formulas

$$\frac{b - c}{a} = \frac{\sin \frac{1}{2}(\beta - \gamma)}{\cos \frac{1}{2}\alpha}$$

$$\frac{c - a}{b} = \frac{\sin \frac{1}{2}(\gamma - \alpha)}{\cos \frac{1}{2}\beta}$$

$$\frac{a - b}{c} = \frac{\sin \frac{1}{2}(\alpha - \beta)}{\cos \frac{1}{2}\gamma}$$

### Newton's Formulas

$$\frac{b + c}{a} = \frac{\cos \frac{1}{2}(\beta - \gamma)}{\sin \frac{1}{2}\alpha}$$

$$\frac{c + a}{b} = \frac{\cos \frac{1}{2}(\gamma - \alpha)}{\sin \frac{1}{2}\beta}$$

$$\frac{a + b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

### Heron's Formula

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

### Other Area Formulas

$$\text{Area} = \frac{bc \sin \alpha}{2} = \frac{ac \sin \beta}{2} = \frac{ab \sin \gamma}{2}$$

$$\begin{aligned} \text{Area} &= \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma} \\ &= \frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta} \\ &= \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} \end{aligned}$$

### Triangle Sides

$$a = b \cos \gamma + c \cos \beta$$

$$b = c \cos \beta + a \cos \alpha$$

$$c = a \cos \alpha + b \cos \gamma$$

### Radius of Inscribed Circle

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}} = \frac{\text{Area}}{s}$$

### Radius of Circumscribed Circle

$$R = \frac{a}{2 \sin \alpha} = \frac{b}{2 \sin \beta} = \frac{c}{2 \sin \gamma} = \frac{abc}{4(\text{Area})}$$

### Angles

$$\sin \alpha = \frac{2}{bc} (\text{Area})$$

$$\cos \alpha = \frac{c^2 + b^2 - a^2}{2bc}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{s(s - a)}{bc}}$$

Analogous formulas hold for other angles.