Integrals of the form $\int \sin^n x \cos^m x \, dx$

1. If the power of $\sin x$ is odd and positive, save one sine factor and convert the remaining factors to cosines. Then let $u = \cos x$.

$$\begin{aligned}
\int \sin^{\overbrace{2k+1}^{\text{Odd}}} x \cos^m x \, dx &= \int \overbrace{(\sin^2 x)^k}^{\text{Convert}} \cos^m x \; \overbrace{\sin x \, dx}^{\text{Save for } du} \\
&= \int (1 - \cos^2 x)^k \cos^m x \sin x \, dx \\
&= -\int (1 - u^2)^k u^m \, du
\end{aligned}$$

2. If the power of $\cos x$ is odd and positive, save one cosine factor and convert the remaining factors to sines. Then let $u = \sin x$.

$$\begin{aligned}
\int \sin^n x \cos^{\overbrace{2k+1}^{\text{Odd}}} x \, dx &= \int \sin^n x \overbrace{(\cos^2 x)^k}^{\text{Convert}} \; \overbrace{\cos x \, dx}^{\text{Save for } du} \\
&= \int \sin^n x (1 - \sin^2 x)^k \cos x \, dx \\
&= \int u^n (1 - u^2)^k \, du
\end{aligned}$$

3. If either the power of $\sin x$ or the power of $\cos x$ is zero, use the guidelines above (if appropriate) or use a power reducing formula. For example

$$\int \sin^4 x \, dx = \int \frac{1}{8} (3 - 4\cos 2x + \cos 4x) \, dx$$

4. If the powers of both $\sin x$ and $\cos x$ are even and nonnegative, use the formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \text{and} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

repeatedly until you can use #2.