

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

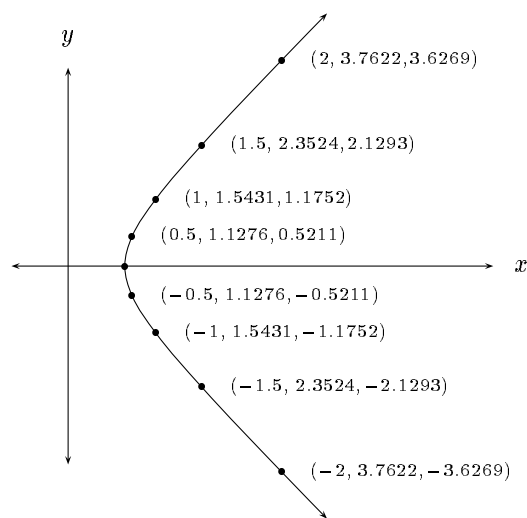
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x}$$

$$\operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{1}{\tanh x}$$



$(t, \cosh t, \sinh t)$

Derivatives

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \operatorname{coth} x = -\operatorname{csch}^2 x$$

$$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \operatorname{coth} x$$

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), \quad (-\infty, \infty)$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), \quad [1, \infty)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right), \quad (-1, 1)$$

$$\operatorname{coth}^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right), \quad (-\infty, -1) \cup (1, \infty)$$

$$\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right), \quad (0, 1]$$

$$\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|} \right), \quad (-\infty, 0) \cup (0, \infty)$$

Hyperbolic Identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

$$\operatorname{coth}^2 x - \operatorname{csch}^2 x = 1$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2}$$

$$\cosh^2 x = \frac{1 + \cosh 2x}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Derivatives

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}} \quad \frac{d}{dx} \operatorname{coth}^{-1} x = \frac{1}{1-x^2}$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}} \quad \frac{d}{dx} \operatorname{sech}^{-1} x = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2} \quad \frac{d}{dx} \operatorname{csch}^{-1} x = \frac{-1}{|x|\sqrt{1+x^2}}$$

Integrals

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left(x + \sqrt{x^2 \pm a^2} \right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 \pm x^2}}{|x|} \right) + C$$