

Limits That Arise Frequently

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 \quad (1)$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt[k]{n}} = 0 \quad (2)$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1 \quad (3)$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n!} = \infty \quad (4)$$

$$\lim_{n \rightarrow \infty} x^{1/n} = 1, \quad (x > 0) \quad (5)$$

$$\lim_{n \rightarrow \infty} x^n = 0, \quad (|x| < 1) \quad (6)$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x, \quad (\text{any } x) \quad (7)$$

$$\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0, \quad (\text{any } x) \quad (8)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (9)$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad (10)$$

$$\lim_{x \rightarrow \infty} \frac{x^p}{e^x} = 0, \quad (\text{any } p) \quad (11)$$

$$\lim_{x \rightarrow \infty} \left(\frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0} \right) = \begin{cases} \pm\infty, & \text{if } m < n \\ a_n/b_m, & \text{if } m = n \\ 0, & \text{if } m > n \end{cases} \quad (12)$$

$$\lim_{n \rightarrow \infty} \frac{(\ln n)^2}{n^a} = 0, \quad (\text{any positive } a) \quad (13)$$